Find the average value of the function on the given interval.

1) $f(x) = x^2, \quad [-1, 1]$

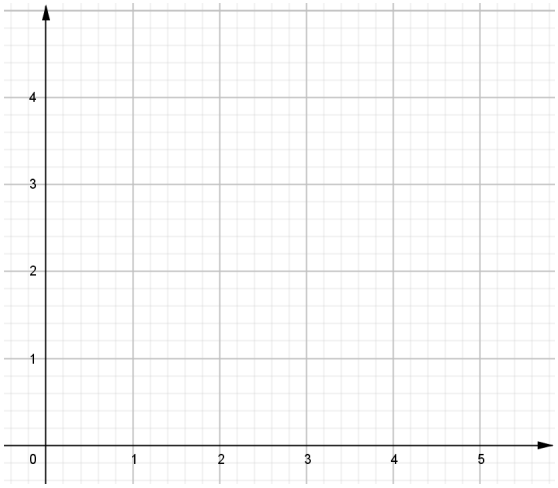
2) $g(x) = x^2\sqrt{1+x^3}, \quad [0, 2]$

3) $f(t) = te^{-t^2}, \quad [0, 5]$

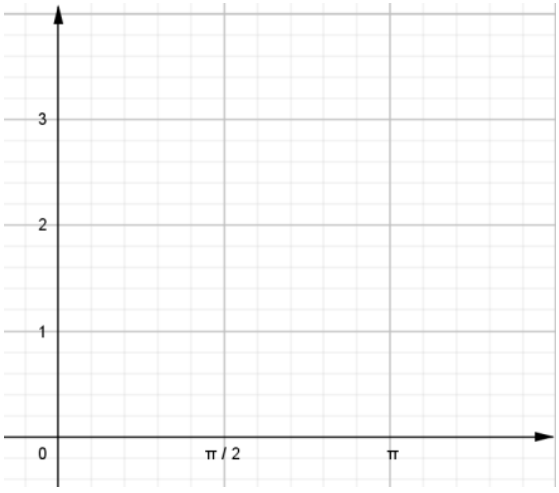
4) $h(x) = \cos^4 x \sin x, \quad [0, \pi]$

Find the average value of f on the given interval. Find c such that $f_{ave} = f(c)$. Sketch the graph f and a rectangle whose area is the same as the area under the graph of f. You may need to use a graphing calculator.

5) $f(x) = (x-3)^2, \quad [2, 5]$



6) $f(x) = 2\sin x - \sin 2x, \quad [0, \pi]$



7) If f is continuous and $\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.

8) Find b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

9) In a certain city the temperature (in °F) t hours after 9 A.M was modeled by the function:

$$T(t) = 50 + 14\sin\frac{\pi}{12}t$$

Find the average temperature during the period from 9 A.M. to 9 P.M.

10) The linear density in a rod 8 m long is $\frac{12}{\sqrt{x+1}}$ kg/m , where x , is measured in meters from one end of the rod. Find the average density of the rod.

11) The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is:

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect to r) over the interval $0 \le r \le R$. Compare the average velocity with the maximum velocity.